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Primordial black holes in braneworld cosmologies: Astrophysical constraints

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In two recent papers we explored the modifications to primordial black hole physics when one moves to the simplest braneworld model, Randall-Sundrum type II. Both the evaporation law and the cosmological evolution of the population can be modified, and additionally accretion of energy from the background can be dominant over evaporation at high energies. In this paper we present a detailed study of how this impacts upon various astrophysical constraints, analyzing constraints from the present density, from the present high-energy photon background radiation, from distortion of the microwave background spectrum, and from processes affecting light element abundances both during and after nucleosynthesis. Typically, the constraints on the formation rate of primordial black holes weaken as compared to the standard cosmology if black hole accretion is unimportant at high energies, but can be strengthened in the case of efficient accretion.

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I. INTRODUCTION

The idea that our observable Universe may be a brane embedded in a higher-dimensional bulk is one which has deep ramifications for cosmology, and which in particular may rewrite many of our ideas as to how the Universe evolved during its earliest stages. In most cosmological contexts, the effects of the braneworld scenario are restricted to early Universe phenomena, though they may impact on present observations by modifying relics from the early Universe, such as inflationary perturbations or the dark matter density. However, in a recent paper ([1], paper I), we reported on an exception to this rule; primordial black holes (PBHs) formed during the early Universe may still probe the bulk dimensions today.

Our analysis was restricted to the simplest braneworld scenario, Randall-Sundrum type II [2] (RS-II), where there is a single bulk dimension of anti-de Sitter form characterized by an AdS radius of curvature l . We showed that provided the AdS radius was sufficiently large, PBHs whose lifetime was as long as the present age of the Universe could behave as five-dimensional objects, and that this would lead to reductions both in the mass and the temperature corresponding to a given lifetime.

In paper I it was argued that at the time a density fluctuation collapses to form a black hole its mass will be of order the horizon mass. Subsequently a braneworld black hole could undergo substantial growth by accreting material from the cosmological background, as shown by Majumdar [3] and by Guedens *et al.* ([4], paper II). This accretion phase will typically last until the standard cosmological regime is

reached, after which the evolution is governed by Hawking evaporation.

Once formed, PBHs will influence later cosmological epochs, leading to a number of observational constraints on their allowed abundance. These have been extensively investigated in the case of the standard cosmology [5–18] (see also Ref. [19] for a review), and the aim of the present paper is to reanalyze the main constraints in the braneworld context. Because the temperatures of black holes evaporating at a given epoch are modified, for the most part such constraints have to be recomputed from first principles.

To outline the types of constraints that can arise, let us consider an epoch labeled by cosmic time t . PBHs whose lifetime exceeds this time will essentially still possess their initial mass and only contribute to the overall energy density. As the observable Universe is close to flatness, a conservative bound derives from the fact that the PBH mass density should not overdominate the Universe. PBHs with lifetimes of order t are evaporating rapidly, producing bursts of evaporation products. Limits can be obtained from imposing they should not interfere disastrously with established processes such as those of nucleosynthesis, or from the fact that these bursts have not been unambiguously observed today [20]. Even shorter-lived PBHs will have evaporated completely at an earlier stage. If this happened well before the decoupling time of a particular species of evaporation product, its Hawking radiation will thermalize with the surroundings, boosting the photon-to-baryon ratio in the process [21]. In the case of evaporation after photon decoupling, the radiation spectrum remains intact and subsequently redshifts. Thus constraints arise from the cosmic background radiation at high frequencies [10–13,20]. Finally, if the radiation is emitted in a certain time window before photon decoupling, it cannot be fully thermalized and will distort the cosmic microwave

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background (CMB) spectrum [22–24]. If it is assumed black holes leave behind a stable relic, this can lead to different constraints [25] but will not be pursued here.

At a given epoch, the impact that is to be constrained is usually dominated by those PBHs with a lifetime of order the cosmic time at the epoch in question. In the accretion phase soon after formation, their energy density will fall off more slowly than that of dust. After the accretion phase we can neglect the energy loss through evaporation until the last stages of their lifetime. Therefore, in the standard cosmological regime the PBHs will predominantly behave similar to dust. We conclude that the fraction of the total energy due to PBHs grows proportional to the scale factor in the radiation-dominated regime, whilst staying constant in the matter-dominated regime. Translating the observational constraint into an upper limit on the initial fraction in PBHs then typically gives extremely strong bounds. Furthermore, since the initial BH mass is of order the horizon mass, the limit on the initial PBH fraction in turn implies a limit on the amplitude of density fluctuations, on scales entering the horizon at the time of formation of the PBH.

In this paper we will reconsider constraints from the present density of black holes with lifetimes exceeding the age of the Universe and from the present photon background, as well as the constraint stemming from the limits on the allowed distortion of the CMB spectrum. We will also reconsider constraints arising from the effect of PBHs evaporating during or after nucleosynthesis on the light element abundances.

II. THE KEY EXPRESSIONS

We begin by reviewing the key results from papers I and II, which can be consulted for the full details. Our Universe is taken to be a flat Friedmann brane, with the effective 4D cosmological constant set to zero. The energy density will be radiation dominated up to the time of matter domination in the more recent past. Under these conditions, there is an early, high-energy regime in which the scale factor, energy density, Hubble radius and horizon mass are given in terms of cosmic time t as

$$a = a_h \left(\frac{t}{t_h} \right)^{1/4}; \quad \rho = \frac{3M_4^2}{32\pi t_c t},$$

$$R_H = 4t; \quad M_H = 8M_4^2 \frac{t^2}{t_c}. \quad (1)$$

This is followed by a standard regime, in which

$$a = a_h \left(\frac{t}{t_h^{1/2} t_c^{1/2}} \right)^{1/2}; \quad \rho = \frac{3M_4^2}{32\pi t^2},$$

$$R_H = 2t; \quad M_H = M_4^2 t. \quad (2)$$

Here, t_h is an arbitrary time in the high-energy regime and $t_c = l/2$ is the transition time between the regimes.

As said, we assume that PBHs form with masses approximately equal to the horizon mass. To incorporate the uncertainty in the nonlinear process of black hole formation, we introduce a factor f as

$$M_i = f M_H(t_i). \quad (3)$$

In the following sections the constraints will be found to be quite insensitive to its precise value.

The main distinction to be made is whether the PBHs are effectively 4D or 5D, which results from comparing the event horizon radius r_0 with the AdS radius l . Interestingly, with $f \lesssim 1$ a PBH will be small ($r_0 \ll l$) if and only if it formed in the high-energy regime. Similarly, it will be large and effectively 4D throughout the bulk of its lifetime if and only if it formed in the standard regime. The behavior of such large black holes should reduce to that of standard cosmology.¹

Neglecting possible charges or rotation, the small PBHs are to good approximation described as 5D Schwarzschild black holes, for which

$$r_0 = \sqrt{\frac{8}{3\pi}} \left(\frac{l}{l_4} \right)^{1/2} \left(\frac{M}{M_4} \right)^{1/2} l_4; \quad (4)$$

$$T_{\text{BH}} = \frac{1}{2\pi r_0}, \quad (5)$$

where an index 4 refers to Planckian quantities as measured on the brane. The evolution in time is obtained from

$$\frac{dM}{dt} = \left(\frac{dM}{dt} \right)_{\text{acc}} + \left(\frac{dM}{dt} \right)_{\text{evap}}, \quad (6)$$

with the accretion and evaporation rates given by

$$\left(\frac{dM}{dt} \right)_{\text{acc}} = \frac{q}{2} \frac{M}{t}, \quad (7)$$

and

$$\left(\frac{dM}{dt} \right)_{\text{evap}} = -\frac{\tilde{g}}{2} \left(\frac{l}{l_4} \right)^{-1} \left(\frac{M}{M_4} \right)^{-1} M_4^2. \quad (8)$$

Following paper II, the factor $q \equiv 4F/\pi$ in the accretion term parametrizes the efficiency with which the black hole accretes the cosmic radiation background, with $F=0$ corre-

¹Recently, an interesting conjecture was made in which the radiation from RS-II black holes can be computed via an AdS conformal field theory (CFT) equivalence [26,27]. If true, this would greatly increase the evaporation rate for large black holes as there are many decay routes into conformal field degrees of freedom, potentially leading to radically different PBH phenomenology. However, their result remains a conjecture, and in this paper we continue to adopt the traditional view of PBH evaporation as described in our earlier papers [1,4]. It would be interesting to fully analyze the modifications to constraints on PBH abundances using their evaporation law.

sponding to no accretion and $F=1$ corresponding to perfect accretion of high-energy radiation from a uniform background. F is therefore expected to lie between zero and one.

In the evaporation rate we have defined

$$\tilde{g} \approx 0.0024 g_{\text{brane}} + 0.0012 g_{\text{bulk}}, \quad (9)$$

where g_{brane} is the usual number of particle species, while $g_{\text{bulk}} = \mathcal{O}(1)$ is the number of bulk degrees of freedom (in the simplest case just the five polarization states of the graviton). The energy lost through Hawking evaporation is mainly emitted onto the brane. As an example we mention the case where the black hole emits only massless particles, for which $g_{\text{brane}} = 7.25$ and $\tilde{g} = 0.023$.

Primordial black holes that are relevant for observational constraints must have lifetimes greatly exceeding the cosmological transition time t_c . It is then an excellent approximation to neglect the evaporation term in Eq. (6) until $t = t_c$, and subsequently to neglect the accretion term. The mass at the transition reads

$$M(t_c) \approx M_i \left(\frac{t_c}{t_i} \right)^{q/2}. \quad (10)$$

We stress that this is the mass the PBH has at the effective onset of evaporation. The total lifetime t_{evap} is given by the 5D mass-lifetime relation

$$\frac{t_{\text{evap}}}{t_4} \approx \tilde{g}^{-1} \frac{l}{l_4} \left(\frac{M(t_c)}{M_4} \right)^2. \quad (11)$$

By assumption $t_{\text{evap}} \gg t_i, t_c$. Therefore, for a PBH of a given lifetime this relation determines what the mass at the onset of evaporation should be, irrespective of the occurrence of accretion. Hence including accretion will not change the expected temperature of PBHs evaporating at a given epoch.

The 5D relations above are to be contrasted with the usual 4D results, in which accretion plays no significant part:

$$r_0 = \frac{2M}{M_4^2}; \quad (12)$$

$$T_{\text{BH}}(4\text{D}) = \frac{M_4^2}{8\pi M}; \quad (13)$$

$$\frac{t_{\text{evap}}(4\text{D})}{t_4} \approx 1.2 \times 10^4 \tilde{g}_{\text{brane}}^{-1} \left(\frac{M_i}{M_4} \right)^3. \quad (14)$$

For black holes of a given lifetime t_{evap} , the question arises if they are effectively 4D or 5D. They will be small (5D) if the AdS radius is greater than a critical value, given by

$$l_{\text{min}}(t_{\text{evap}}) = \tilde{g}^{1/3} \left(\frac{t_{\text{evap}}}{t_4} \right)^{1/3} l_4. \quad (15)$$

For much smaller values of l , the standard 4D case is retrieved. For example, all PBHs with lifetimes up to the present age of the Universe would have been five dimensional throughout their evolution provided that $l > 10^{20} l_4$.

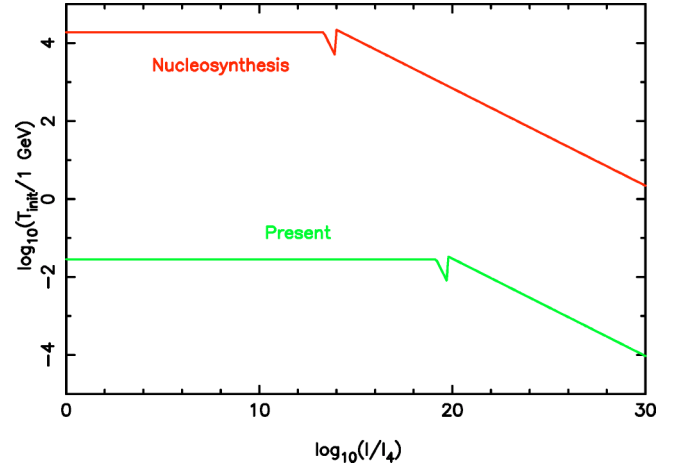


FIG. 1. The initial temperature of black holes evaporating at the key epochs of nucleosynthesis ($t_{\text{evap}} \approx 100$ s) and the present ($t_{\text{evap}} \approx 14$ Gyrs), shown as a function of l .

As the experimental upper limit on the AdS radius currently is quite weak ($l < l_{\text{max}} \approx 10^{31} l_4$ [28]), there is considerable room for 5D PBHs to play a role in the cosmological history, including the present.

A simple reworking of the 5D relations expresses the BH mass and temperature at the onset of evaporation in terms of l and t_{evap} :

$$\frac{M(t_c)}{M_4} = \tilde{g}^{1/2} \left(\frac{t_{\text{evap}}}{t_4} \right)^{1/2} \left(\frac{l}{l_4} \right)^{-1/2}, \quad (16)$$

$$\frac{T_{\text{BH}}}{T_4} = \sqrt{\frac{3}{32\pi}} \tilde{g}^{-1/4} \left(\frac{t_{\text{evap}}}{t_4} \right)^{-1/4} \left(\frac{l}{l_4} \right)^{-1/4}. \quad (17)$$

The temperature of PBHs evaporating today and at nucleosynthesis is shown as a function of l in Fig. 1. Plots of the mass are qualitatively the same. At the smallest values of l , the temperature assumes the 4D value, for $l \approx l_{\text{min}}$ (which is equivalent to $r_0 \approx l$) accurate description is uncertain through lack of exact solutions, while for larger l values the BH temperature is reduced. Since most of the energy of a PBH is radiated at temperatures close to the start temperature at the onset of evaporation, we conclude that the evaporation products present at a certain epoch in cosmology will be cooler if l is sufficiently large. For example, PBHs evaporating today would produce no massive particles, except in a high-energy tail from the late stages of evaporation.

To conclude, we list the formation times t_i in terms of a given lifetime t_{evap} . For a PBH that formed in the high-energy regime this is

$$\begin{aligned} \left(\frac{t_i}{t_4} \right)_{\text{HE}} &= \frac{1}{4} f^{-1/2} \left(\frac{l}{l_4} \right)^{1/2} \left(\frac{M_i}{M_4} \right)^{1/2} \\ &= \left[2^{q-8} \frac{\tilde{g}}{f^2} \left(\frac{l}{l_4} \right)^{1-q} \left(\frac{t_{\text{evap}}}{t_4} \right) \right]^{1/(4-q)}, \end{aligned} \quad (18)$$

while in the standard regime one finds

$$\left(\frac{t_i}{t_4}\right)_{\text{ST}} = f^{-1} \frac{M_i}{M_4} = 0.04 f^{-1} g_{\text{brane}}^{1/3} \left(\frac{t_{\text{evap}}}{t_4}\right)^{1/3}. \quad (19)$$

It is worth bearing in mind that if black holes form after inflation, there is an upper limit on the mass scale coming from gravitational wave production. In paper I we showed this gives a lower limit on the black hole mass at formation of

$$M_i > 2 \times 10^6 M_5. \quad (20)$$

Using the lifetime-formation time relation Eq. (18), Eq. (20) implies a lower limit on the black hole's lifetime:

$$\frac{t_{\text{evap}}}{t_4} > \frac{f^2}{g} 2^{2+q/2} \times 10^{3(4-q)} \left(\frac{l}{l_4}\right)^{(2q+1)/3} \equiv \frac{t_{\text{evap, min}}}{t_4}. \quad (21)$$

For $l = 10^{31} l_4$ and 100% efficiency this gives

$$t_{\text{evap}} > 1100 f^2 \text{ s}, \quad (22)$$

i.e. no black holes evaporating until after nucleosynthesis. But for an efficiency lower than 86% or when $l/l_4 < 10^{28}$ the lightest black holes allowed by the lower mass limit can evaporate before $t = 1$ s. Note however that the above limit may be a very conservative one. If inflation was at an energy scale much lower than the allowed upper limit, the lightest permitted PBHs may evaporate at much later times still.

III. CONSTRAINT FORMALISM

Constraints on the allowed abundance of PBHs of a certain lifetime are formulated as upper bounds on their mass fraction. This mass fraction $\alpha_i(M_i)$ will be defined as the ratio of the energy density due to PBHs of initial mass M_i and the background radiation density, at a time $t \geq t_i$:

$$\alpha_i(M_i) \equiv \frac{\rho_{\text{PBH}, M_i}(t)}{\rho_{\text{rad}}(t)}. \quad (23)$$

The initial and final mass fractions will be denoted α_i and α_{evap} respectively. The purpose of the following sections is to reconsider observational constraints on α_{evap} at different cosmological epochs, trace them back to obtain constraints on the initial mass fraction α_i , and to compare both types of constraint in the standard and braneworld cosmologies. Let us consider a constraint imposed at a given epoch of cosmic time τ :

$$\alpha_i < L_\tau^{\text{ST}} \quad \text{or} \quad \alpha_i < L_\tau^{\text{HE}}, \quad (24)$$

in the standard or braneworld scenario respectively. Several constraints imposed at this stage do not depend on the individual temperature or mass of the PBHs, but only on the total

energy contained in them or emitted by them, so that $L_\tau^{\text{ST}} = L_\tau^{\text{HE}}$. Examples include the bound from the present mass density, from the distortion of the CMB, or the deuterium photodisintegration constraint. An example of a bound that does depend on the individual characteristics of the PBH is the helium abundance constraint (see Sec. VII A).

To obtain the corresponding bound on the initial mass fraction requires knowledge of the cosmic evolution. Note that in the high-energy regime, i.e., in the PBH accretion phase, we have

$$\alpha_i \propto M(t) a(t), \quad (25)$$

whereas in the standard regime the mass fraction simply grows with the scale factor (until the final epoch of the PBH's lifetime). Tracing Eq. (24) back to the time of formation, we get

$$\alpha_i < L_\tau^{\text{ST}} \left(\frac{a_i}{a_\tau}\right)_{\text{ST}} \equiv L_i^{\text{ST}} \quad (26)$$

or

$$\alpha_i < L_\tau^{\text{HE}} \frac{M_i}{M(t_c)} \left(\frac{a_i}{a_\tau}\right)_{\text{HE}} \equiv L_i^{\text{HE}}. \quad (27)$$

In order to compare the initial constraints Eqs. (26) and (27), we express their ratio as

$$\frac{L_i^{\text{HE}}}{L_i^{\text{ST}}} = \frac{L_\tau^{\text{HE}}}{L_\tau^{\text{ST}}} \frac{M_i}{M(t_c)} \frac{(a_i)_{\text{HE}}}{(a_i)_{\text{ST}}}. \quad (28)$$

The ratio of the initial scale factors can be expressed as

$$\frac{(a_i)_{\text{HE}}}{(a_i)_{\text{ST}}} = \frac{t_{i,\text{HE}}^{1/4} t_c^{1/4}}{t_{i,\text{ST}}^{1/2}}. \quad (29)$$

As mentioned in the Introduction, the impact that is to be constrained at a certain epoch is usually dominated by PBHs with lifetimes of order the cosmic time at that epoch. We therefore take $\tau = t_{\text{evap}}$. Using Eq. (10) for $M(t_c)$, and Eqs. (18) or (19) for the lifetime-formation time relations, we obtain³

$$\frac{L_i^{\text{HE}}}{L_i^{\text{ST}}} \approx \frac{L_{\text{evap}}^{\text{HE}}}{L_{\text{evap}}^{\text{ST}}} \left(\frac{l}{l_{\text{min}}}\right)^{(5-8q)/4(4-q)}, \quad (30)$$

where l_{min} is the value of the AdS radius at which quantities obtained in the HE scenario reduce to the standard ones, see Eq. (15). This equation is illustrated in Fig. 2. Since $l > l_{\text{min}}$, which constraint is the stronger is simply determined

²The reader is warned that different notations are sometimes used, such as $\alpha = \rho_{\text{pbh}}/\rho_{\text{tot}}$. This will not be important in the radiation-dominated phase, but the definitions do deviate in the matter-dominated phase.

³We have omitted powers of f , the ratio of the initial black hole mass and the horizon mass [see Eq. (3)]. Taking them into account we find $L_i^{\text{HE}} \propto f^{-(1+2q)/2(4-q)}$ and $L_i^{\text{ST}} \propto f^{-1/2}$. Since the accretion parameter q is not expected to be larger than 1.5, these factors are of peripheral significance.

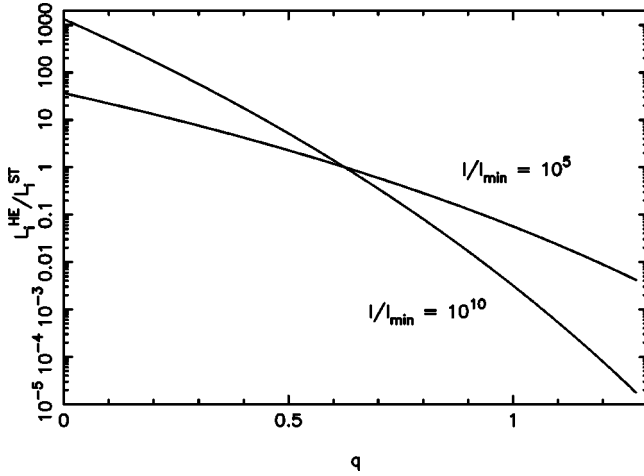


FIG. 2. The behavior of Eq. (30) as a function of the accretion parameter q for two choices of the ratio l/l_{\min} , under the assumption that the imposed limit at evaporation is identical in the high energy or standard treatment, i.e. $L_{\text{evap}}^{\text{HE}} = L_{\text{evap}}^{\text{ST}}$. This shows how constraints are modified compared to the standard cosmology in situations where the constraint at evaporation is unchanged. If accretion is negligible the constraints are weakened in the high-energy case, whereas for $q > 5/8$ accretion leads to the constraints strengthening in the braneworld case.

by the sign of $(5 - 8q)$, at least when $L_{\text{evap}}^{\text{ST}} = L_{\text{evap}}^{\text{HE}}$. Thus the initial constraint will be stronger in the braneworld scenario if the accretion efficiency F is more than 49%, whilst becoming weaker for efficiencies below 49%. Whatever the accretion efficiency, maximum discrepancy with the standard constraint is found when $l = l_{\max} \approx 10^{31} l_4$. Since $l_{\min} \propto t_{\text{evap}}^{1/3}$, this maximum discrepancy rises mildly when considering earlier epochs, i.e., shorter-lived PBHs.

Note that if the calculations of paper II turn out to be invalid, and accretion is in fact not an important process even during the high-energy regime, the correct constraints are retrieved by filling in $q = 0$ in the above formalism.

IV. PBHs MUST NOT OVERDOMINATE THE UNIVERSE

For a particular value of l , Eq. (18) or (19) with $t_{\text{evap}} = t_0 = 8 \times 10^{60} t_4$ gives the formation time of PBHs that are evaporating today. Black holes formed later are essentially still intact. Their density is constrained by the observed matter density in the present Universe; specifically, we are tracking the relative densities of PBHs to radiation, and we must ensure that, given the observed radiation density, this ratio does not imply that the PBH density exceeds the observed matter density of about 0.3 of the critical density. Phrased in this way, the constraint applies regardless of the presence of a cosmological constant, and indicates that for any PBHs surviving to the present we must have

$$\alpha_0(M) < \frac{0.3}{\Omega_{\gamma,0}}. \quad (31)$$

The cosmic microwave background corresponds to a photon density as $\Omega_{\gamma,0} h^2 = 2.47 \times 10^{-5}$, with $h \approx 0.7$, and conservatively we can ignore the cosmic neutrinos.

From Eqs. (26) and (27) it is clear that a given observational constraint will give the most severe initial constraint for the lightest PBHs it applies to, i.e. those that formed the earliest. Employing Eq. (31) for PBHs that are about to evaporate today, $t_{\text{evap}} \approx t_0$, we find

$$\alpha_{\text{evap}} < \frac{0.3}{\Omega_{\gamma,0}} \approx 6 \times 10^3 \equiv L_{\text{evap}}. \quad (32)$$

In the standard cosmology this is a bound on PBHs with mass $M_i \approx 2 \times 10^{19} M_4 = 4 \times 10^{14} \text{g}$, and the constraint on the initial PBH mass fraction, Eq. (26), reads [9,14]

$$\alpha_i < f^{-1/2} \times 10^{-18}. \quad (33)$$

As examples in the braneworld scenario, we consider the case of maximum AdS radius, without accretion or with 100% accretion respectively, i.e. the cases $l = l_{\max}$, $q = 0$, and $l = l_{\max}$, $q = 4/\pi$, and make use of Eq. (30). The first example gives $M_i = 10^{14} M_4 = 3 \times 10^9 \text{g}$ and

$$\alpha_i < f^{-1/8} 10^{-14}, \quad (34)$$

while the second example results in $M_i = 6 \times 10^5 M_4 = 10 \text{g}$ and

$$\alpha_i < f^{-0.65} 10^{-23}. \quad (35)$$

These are examples of the general trend discussed earlier, that going to the braneworld case without considering accretion weakens the constraint on the formation rate, but that including accretion can strengthen it, with fully efficient accretion leading to a more powerful constraint than in the standard cosmology.

V. THE PRESENT PHOTON SPECTRUM

If PBHs evaporate between the time of photon decoupling ($t_{\text{dec}} \approx 10^{12} \text{s}$) and the present day, their radiation spectra will not be appreciably influenced by the background Universe, apart from being redshifted. Thus the spectra could constitute a fraction of the cosmic background radiation. We will restrict attention to the photon spectrum. In Refs. [29,20] it was pointed out that black holes with temperatures above the QCD scale (200–300 MeV) should emit quark and gluon jets that subsequently fragment into particles whose rest mass is below the black hole temperature. This could significantly alter the spectrum as compared to when the particles are emitted directly as Hawking radiation. However, for large values of the AdS radius, the temperature of PBHs evaporating after decoupling will never be above a few MeV, in which case the effect should be negligible. We derive an expression for the spectral shape, and calculate it explicitly in the assumption of a scale invariant initial PBH spectrum. We then consider the constraint to be imposed on the peak of the spectrum.

A. The spectral shape

The spectral photon number emitted onto the brane by a small black hole with lifetime t_{evap} is obtained from

$$\frac{dN}{dE} = \int_0^{t_{\text{evap}}} \frac{\sigma(E)}{2\pi^2} \frac{E^2}{\exp[E/T_{\text{BH}}(t)] - 1} dt, \quad (36)$$

with $\sigma(E)$ the emission cross-section for photons of frequency $\omega = E$. In terms of the integration variable

$$x \equiv \frac{E}{T_{\text{BH}}(t)}, \quad (37)$$

and using the 5D relations of Sec. II, this becomes

$$\frac{dN}{dE} = \frac{9}{512 \pi^4 \tilde{g}} \left(\frac{l}{l_4}\right)^{-1} M_4^3 E^{-2} \int_0^{x_i} \sigma(x) \frac{x^3}{e^x - 1} dx, \quad (38)$$

where $x_i = E/T_{\text{BH}}$ and T_{BH} denotes the temperature of the black hole at the onset of evaporation. In the high-frequency limit ($E \gg T_{\text{BH}}$), all cross sections reduce to the same value (see paper I)

$$\sigma = 4\pi r_0^2 = \frac{1}{\pi} E^{-2} x^2. \quad (39)$$

In this limit, the spectral number becomes

$$\frac{dN}{dE} = \frac{\pi}{448} \tilde{g}^{-1} \left(\frac{l}{l_4}\right)^{-1} M_4^3 E^{-4}. \quad (40)$$

Note that the spectrum declines as E^{-4} , compared to the E^{-3} tail of the standard spectrum.

Now consider black holes evaporating at a time $t_{\text{evap}} \gg t_{\text{dec}}$. We will make the approximation that all the energy gets released instantly, but will take the spectrum into account. The black hole mass fraction just before evaporation is given by

$$\alpha_{\text{evap}} = \alpha_i \frac{M(t_c)}{M_i} \frac{a(t_{\text{evap}})}{a(t_i)}, \quad (41)$$

while the number density is

$$n_{\text{PBH}}(t_{\text{evap}}) = \alpha_{\text{evap}} \frac{\rho_{\text{rad}}(t_{\text{evap}})}{M(t_c)}. \quad (42)$$

The energy density in photons of energy E , emitted between t_{evap} and $t_{\text{evap}} + dt_{\text{evap}}$ is

$$dU_{t_{\text{ev}}}(E) \equiv n_{\text{PBH}}(t_{\text{evap}}) E^2 \frac{dN}{dE}(t_{\text{evap}}) \frac{dt_{\text{evap}}}{t_{\text{evap}}}. \quad (43)$$

We require the present total energy density in Hawking photons at a certain energy scale E_0 , denoted as $U_0(E_0)$. Radiation emitted at t_{evap} with energy E will be redshifted to E_0 today provided

$$E(E_0) = E_0 \frac{a(t_0)}{a(t_{\text{evap}})}. \quad (44)$$

Integrating over all times t_{evap} after decoupling, we have

$$U_0(E_0) \equiv \int_{t_{\text{dec}}}^{t_0} dU_0(E_0) = \int \frac{a(t_{\text{evap}})^4}{a(t_0)^4} dU_{t_{\text{ev}}}[E(E_0)]. \quad (45)$$

We will assume $t_i(t_0) < t_c$, i.e. all PBHs light enough to have evaporated completely today were formed in the high-energy regime. Substituting the relevant formulas then results in⁴

$$U_0(E_0) = \frac{8^{1/4}}{6\pi} \tilde{g}^{-9/16} \left(\frac{t_0}{t_4}\right)^{-4/3} \left(\frac{t_{\text{eq}}}{t_4}\right)^{1/2} \left(\frac{l}{l_4}\right)^{3/16} M_4^3 E_0^2 \times \int \alpha_i \left(\frac{t_{\text{evap}}}{t_4}\right)^{-59/48} \frac{dN}{dE(E_0)}(t_{\text{evap}}) Q(t_{\text{evap}}) \frac{dt_{\text{evap}}}{t_{\text{evap}}}, \quad (46)$$

with the factor $Q(t_{\text{evap}})$ incorporating the effect of accretion. It is defined by

$$Q(t_{\text{evap}})^{4(4-q)/q} = 2^9 \tilde{g}^{-9/4} \left(\frac{l}{l_4}\right)^{27/4} \left(\frac{t_{\text{evap}}}{t_4}\right)^{-9/4} \quad (47)$$

and reduces to $Q = 1$ if accretion is neglected.

The number spectrum of a black hole of initial temperature T_{BH} peaks at an energy $E = b T_{\text{BH}}$, with $b \approx 5$ in the standard treatment [30]. Therefore, unless α_i is sharply peaked at particular initial epochs, the main contribution to the integral in Eq. (46) is obtained when $E(E_0) = b T_{\text{BH}}(t_{\text{evap}})$, i.e., from PBHs evaporating at $t_{\text{evap}} = t_{\text{main}}$, where

$$t_{\text{main}} \approx \left(\frac{E_0}{b T_{\text{BH}}(t_0)}\right)^{12/5} t_0. \quad (48)$$

The contribution from PBHs evaporating earlier will come from the high-frequency end of their spectrum, while PBHs evaporating at later times will contribute radiation that originated in the low-frequency end. Using the number spectrum Eq. (38) with $x_i = b$, we estimate the total energy density at energy E_0 as

$$U_0(E_0) \approx 0.1 \alpha_i \tilde{g}^{-7/10} b^{-69/20} \left(\frac{t_{\text{eq}}}{t_4}\right)^{1/2} \left(\frac{t_0}{t_4}\right)^{-17/10} \left(\frac{l}{l_4}\right)^{1/20} \times \left(\frac{E_0}{M_4}\right)^{29/20} Q(t_{\text{main}}) M_4^4, \quad (49)$$

with

$$Q(t_{\text{main}})^{4(4-q)/q} = 2^9 \pi^{-27/10} b^{27/5} \tilde{g}^{-18/5} \left(\frac{l}{l_4}\right)^{27/5} \left(\frac{t_0}{t_4}\right)^{-18/5} \times \left(\frac{E_0}{M_4}\right)^{-27/5}. \quad (50)$$

The shape of the spectrum is seen to depend on the accretion parameter q . Introducing p through $U_0(E_0) \propto E_0^p$, the expo-

⁴Having omitted powers of f as discussed in previous sections.

nent ranges from $p = 1.45$ for $q = 0$ to $p = 0.82$ for $q = 4/\pi$. For comparison, in the standard scenario we have $p = 1.5$ in this range of the spectrum. The time t_{main} as defined above will lie in the relevant time interval only for energies E_0 in the interval

$$\left(\frac{t_{\text{dec}}}{t_0}\right)^{5/12} b T_{\text{BH}}(t_0) < E_0 < b T_{\text{BH}}(t_0). \quad (51)$$

Radiation at lower frequencies will stem completely from the low-frequency ends of the instantaneous spectra, with the dominant contribution coming from PBHs evaporating around t_{dec} . Its intensity can generically be neglected as compared to the main frequency range.⁵ For energies $E_0 > b T_{\text{BH}}(t_0)$, the dominant part comes from the high-frequency tail of PBHs evaporating today. The number spectrum Eq. (38) with $x_i = \infty$ is used to obtain

$$U_0(E_0) \approx 10^{-3} \alpha_i \tilde{g}^{-25/16} \left(\frac{t_{\text{eq}}}{t_4}\right)^{1/2} \left(\frac{t_0}{t_4}\right)^{-41/16} \left(\frac{l}{l_4}\right)^{-13/16} Q(t_0) \times \left(\frac{E_0}{M_4}\right)^{-2} M_4^4. \quad (52)$$

B. The observational constraint

To make contact with observations, it is convenient to relate the integrated energy density $U_0(E_0)$ to the spectral surface brightness $I(E_0)$ through

$$I(E_0) = \frac{c}{4\pi} \frac{U_0(E_0)}{E_0}. \quad (53)$$

The overall peak in the present spectrum is at $E_{\text{peak}} = b T_{\text{BH}}(t_0)$. Using $q = 0$, $b = 5$, $t_0 = 8 \times 10^{60} t_4$, $t_{\text{eq}} = 10^{-6} t_0$ and $g = 0.023$, we find for $I(E_{\text{peak}})$, expressed in the units $\text{keV cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{keV}^{-1}$:

$$I(E_{\text{peak}}) \approx 10^{23} \alpha_i \left(\frac{l}{l_4}\right)^{-1/16}. \quad (54)$$

The strongest constraint to be placed on the PBH spectrum from the observed flux arises at $E = E_{\text{peak}}$. Employing the 5D relations of Sec. II for $t_{\text{evap}} = t_0$, we find that E_{peak} can range from a hundred MeV to a few hundred keV, depending on the AdS radius. Thus if the AdS radius is large, PBHs would mainly contribute to the hard x-ray background (XRB) [31–33]. The XRB exhibits a peak at around 30 KeV, thought to be sourced by AGN [34], although the issue is not settled at present. This peak arguably is at too low an energy scale to be explained by braneworld PBHs, but note that $T_{\text{BH}}(t_0) \approx 50 \text{ keV}$ for $l = l_{\text{max}}$ in our simple setup. There was long thought to be another excess of radiation at 1 to 10 MeV (the so-called “MeV bump”) [35,31]. However, measurements made with the COMPTEL telescope [36] have shown this to be due to an instrumentation error, and the background is

⁵In addition, the Universe is still rather opaque at these early times.

now believed to be smooth over a wide range of energies. From the steepness of the PBH spectrum Eqs. (49) and (52) we conclude that, if there is a spectrum of radiation due to evaporating PBHs, it should be significantly fainter than the observed background.

As an example we consider $l = l_{\text{max}}$, to give a peak energy $E_{\text{peak}} \approx 250 \text{ keV}$. The observed surface brightness at this energy is given by [33]

$$I_{\text{obs}} = 0.1 \text{ keV cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{keV}^{-1}. \quad (55)$$

The constraint $I(E_{\text{peak}}) < I_{\text{obs}}$ then results in an upper limit on the initial mass fraction

$$\alpha_i < 10^{-23}. \quad (56)$$

For comparison, the corresponding constraint in the standard case is obtained from the gamma-ray background at $E_{\text{peak}} \approx 100 \text{ MeV}$ and reads $\alpha_i < 10^{-27}$ [10–13,18].

An equivalent way to constrain the peak value of the photon spectrum is found by expressing the observed density of radiation of order a hundred keV in terms of its density parameter as

$$\Omega \approx 10^{-9}. \quad (57)$$

Assuming that the fraction of the PBH mass going into photons is roughly ten percent, one obtains

$$\Omega_{\text{pbh}} < 10^{-8}, \quad (58)$$

which is identical in form to the constraints of Sec. IV, and simply strengthens them by 8 orders of magnitude.

VI. DISTORTION OF THE COSMIC MICROWAVE BACKGROUND SPECTRUM

Energy that is released at a time $t_{\text{SZ}} \approx 10^{-10} t_0$ will fail to thermalize fully with the background radiation, modifying the Planck law with a chemical potential μ [23,22]. The injected energy ΔE is related to μ as

$$\frac{\Delta E}{E} = 0.71 \mu, \quad (59)$$

with E the background energy. Observational results [22] suggest an upper limit on μ given by

$$\mu < 9 \times 10^{-5}. \quad (60)$$

This can be used to constrain the fraction of PBHs evaporating around the time t_{SZ} , as will now be elaborated. Since the period under consideration occurs well after neutrino decoupling, only the fraction F of the PBH energy released onto the brane that does not go into neutrinos (or gravitons) will be relevant for the above bound. For the largest values of the AdS radius, the temperature of the black holes evaporating at t_{SZ} is such that the electron is the only massive particle to be produced, leading to an estimate for F as

$$F \approx 0.5. \quad (61)$$

Smaller values of the AdS radius presumably increase F somewhat, but this will not play a substantial part in our order of magnitude estimate of the constraint. When the injected energy derives solely from black hole evaporation, Eq. (59) becomes

$$F \alpha_{\text{evap}}(t_{\text{SZ}}) = 0.71 \mu. \quad (62)$$

Translating the observational bound Eq. (60) into a bound on the mass fraction of PBHs evaporating around the time t_{SZ} results in

$$\alpha_{\text{evap}} < 1.3 \times 10^{-4} \equiv L_{\text{evap}}. \quad (63)$$

For the standard cosmology this gives an initial constraint

$$\alpha_i < 10^{-21}, \quad (64)$$

as first obtained in Ref. [24]. The corresponding initial limits in the braneworld scenario for the example of $l = l_{\text{max}}$, are $L_i^{\text{HE}} = 10^{-17}$ for $q = 0$ and $L_i^{\text{HE}} = 10^{-28}$ for $q = 4/\pi$.

VII. NUCLEOSYNTHESIS CONSTRAINTS

In principle every cosmological era prior to the time of photon decoupling, at around 10^{12} seconds, could have been affected by PBH particle interactions [11]. Of these, the era of standard big-bang primordial nucleosynthesis (SBBNS) is generally regarded as being the best understood and the most tightly constrained, and so presents the best place in which to look for such effects. To that end, a number of detailed investigations have already provided strong evidence to suggest that a range of nucleosynthesis reactions and parameters should indeed have been modified in the presence of evaporating PBHs, and that furthermore, for a sufficiently high density of PBHs, such modifications would ultimately have led to changes in the final light element abundances [37–43]. Broadly speaking, in this body of work existing observational limits on the light element abundances are used to put constraints on the size of such modifications, which then typically lead to strong constraints on the numbers of PBHs allowed to evaporate both during and after nucleosynthesis. Here we reexamine two nucleosynthesis constraints in the context of the braneworld, namely the constraint on the increase in production of helium-4 due to the injection of PBH hadrons [38], and the constraint on the destruction of primordial deuterium by PBH photons [41]. Our main aim here is to get a reasonable estimate of how such constraints are modified, and not to perform a quantitatively precise calculation.

From Eq. (14), one finds that in the standard scenario, PBHs which evaporate during nucleosynthesis, which we shall take to be between ~ 1 and 400 s, have initial masses which range from $\sim 10^9$ g at 1 s, to $\sim 10^{10}$ g at 400 s, with corresponding temperatures that range from $\sim 10^4$ to $\sim 10^3$ GeV. On the other hand, in the braneworld picture we find from Eqs. (16) and (17) that PBHs which decay during nucleosynthesis have masses at the effective onset of evaporation in the range

$$\tilde{g}^{1/2} \left(\frac{l}{l_4} \right)^{-1/2} \leq \frac{M(t_c)}{10^{17} \text{ g}} \leq 20 \tilde{g}^{1/2} \left(\frac{l}{l_4} \right)^{-1/2} \quad (65)$$

with corresponding temperatures in the range

$$5 \tilde{g}^{-1/4} \left(\frac{l}{l_4} \right)^{-1/4} \geq \frac{T_{\text{BH}}}{10^7 \text{ GeV}} \geq \tilde{g}^{-1/4} \left(\frac{l}{l_4} \right)^{-1/4}. \quad (66)$$

Strictly speaking, since \tilde{g} is temperature dependent, the temperature of the evaporating braneworld PBH has to be solved iteratively. However, since the temperature dependence of \tilde{g} is in fact very weak over the range of interest, for most purposes an estimate of $\tilde{g} \sim 0.1$ usually suffices for all temperatures up to a few TeV or so.⁶ Accordingly, for the maximum value of l allowed by present observational limits $l \approx 10^{31} l_4$, the PBHs evaporating during nucleosynthesis have masses which range from ~ 10 g at 1 s to ~ 200 g at 400 s and have corresponding temperatures which range from around 1 to 0.2 GeV. Thus, we may conclude that at the limit of the largest l currently allowed, the “small” PBHs evaporating during nucleosynthesis can evaporate a wide range of particles, e.g. massless particles, neutrinos, electrons and positrons, muons, pions, etas, kaons, and in addition, they are just hot enough to evaporate nucleons.⁷ At the other extreme we can take the smallest value of l for which Eqs. (65) and (66) are valid, i.e. $l = l_{\text{min}}$. Then the temperature and mass range of the standard scenario are retrieved, as they must do.

A. The helium abundance constraint

The proton-to-neutron ratio, n/p , is a key parameter in primordial nucleosynthesis (for reviews see Refs. [44–50]) and small variations in its size can have appreciable effects on the final values of the light element abundances, ^4He , ^3He , D , ^7Li , that the theory predicts.

According to the standard picture, n/p “froze out,” i.e. fell out of equilibrium and became fixed at a nearly constant value, at around 1 s, when the weak interaction proton-neutron interconversion rate, $\Gamma_{n \leftrightarrow p} \sim G_F^2 T^5$, fell below the Hubble expansion rate $H \sim (G_N g_*)^{1/2} T^2$. Since n/p had been kept near Boltzmann equilibrium up until this time, at freeze out it would have had a value of $\approx \exp(-\Delta M_{np}/T_f) \approx 1/6$, where $T_f \approx 0.8$ MeV was the temperature at freeze-out

⁶For standard model fields on the brane and gravity in the bulk g_{brane} and \tilde{g} respectively range from around 7.25 and 0.023 , for $T \lesssim 1$ MeV to about 106.75 and 0.26 for $300 \text{ GeV} \lesssim T \lesssim 1 \text{ TeV}$. Therefore, over the temperature range $1 \text{ MeV} \lesssim T \lesssim 1 \text{ TeV}$, we have $0.15 \lesssim \tilde{g}^{1/2} \lesssim 0.51$ (factor of 4) and $2.9 \gtrsim \tilde{g}^{-1/4} \gtrsim 1.4$ (factor of 2). Hence, it is good enough in most calculations to take $\tilde{g} \sim 0.1$. In principle, however, we note that \tilde{g} could take a wider range of values at temperatures much higher than the TeV scale.

⁷With regard to the evaporation of nucleons, it should be also noted that, in taking proper account of the gray body factors, it is expected that the actual Hawking temperature of these black holes will be hotter by a factor of a few than the pure black body temperatures quoted here.

and $\Delta M_{np} \sim 1.293$ MeV the neutron-proton mass difference. Subsequently the value of n/p was then affected only by neutron beta decay. In addition, however, neutrons and protons at this time were also undergoing collisions and were thereby able to form deuterium, via reactions such as $p + n \rightleftharpoons D + \gamma$. Initially, however, the energy and density of the photon background was sufficiently high so as to photodissociate all of the D that formed in this way. Only after the temperature of the Universe had fallen below about 0.8 KeV, which occurred at a time of around 100 seconds, did the photodissociation rate drop below the D binding rate and nucleosynthesis start. Once this so-called “D bottleneck” had been breached, the binding of protons and neutrons into D was then quickly followed by the subsequent binding of D with protons and neutrons into tritium and ^3He nuclei, and in turn the binding of these into ^4He . However, because of the absence of both stable mass-five and mass-eight nuclei, and in addition the existence of strong Coulomb barriers to all reactions that could form nuclei with mass-six, seven, nine or heavier, these nucleosynthesis reactions were only able to proceed efficiently as far as ^4He . Consequently, the process essentially ended once all of the neutrons (i.e., the fuel) had been bound into ^4He nuclei, leaving only a very small proportion bound in the form of D and ^3He , together with an even smaller proportion that were able to overcome the Coulomb barriers and go on further to bind into heavier nuclei, mainly ^7Li and ^7Be .

The production of ^3He and D was so-called *rate limited*, i.e. the precise quantities of these nuclei left over at the end of nucleosynthesis were determined by the efficiency of their binding into ^4He , or in other words by the reaction rates of the associated binding processes. Since the reaction rates were simply proportional to the values of the speed of light, the thermally averaged cross sections and the density of the baryons, i.e., $\Gamma_a \sim c \langle \sigma, T \rangle n_a$, the efficiency of each process would have been sensitive to a variation in any of these quantities. Hence, in the absence of non-standard physics affecting the cross sections, the determining factor would have been the baryon density, n_b .⁸ Thus, the predictions of nucleosynthesis calculations are a function of essentially just one parameter, namely n_b , although this is usually expressed in terms of its photon number density normalized form, η . The greater the density of baryons, i.e. the higher the value of η , the faster the reaction rates and the more efficient and complete the binding of neutrons into ^4He and so the less left behind in the form of ^3He and D and vice versa.

The production of the heavier nuclei in nucleosynthesis are likewise sensitive to η , but display a more complex dependence on its value. By the time nucleosynthesis actually started at 100 s, β -decay had lowered n/p to around 1/7. Given that nucleosynthesis ended once virtually all of the neutrons had been bound into ^4He , it therefore follows that the *mass fraction* of ^4He at the end of nucleosynthesis,

$Y_p = 4n_{^4\text{He}}/n_b$, should have been approximately twice the value of the neutron-to-baryon ratio when it began: namely,

$$Y_p \approx \frac{2n}{n+p} = \frac{2n/p}{1+n/p} \approx 0.25. \quad (67)$$

Although this estimate may seem somewhat crude, it is in fact to first order in good agreement with a full numerical treatment and also with the current observational bounds. The production of ^4He has only a mild sensitivity to η , since it is guaranteed that virtually all of the neutrons will end up in ^4He whatever else happens. The small dependence that it does display derives from the fact that higher η values allow earlier D-bottleneck breaching times, implying that nucleosynthesis can start sooner and therefore with higher initial values of n/p . Aside from the η dependence, the actual uncertainty that currently arises in the *theoretical* prediction of Y_p , is of order 0.2% ($\sigma_Y = 0.0005$) [51], and comes primarily from the (now small) uncertainty in the neutron lifetime [45], which is presently estimated to be $\tau_n = 885.7 \pm 0.8$ s [52].

During the intervening period between the end of nucleosynthesis and the present, the universal light element abundances are *all* believed to have undergone a certain amount of chemical evolution due to the effects of stellar processing. In order to estimate the primordial abundances today, therefore, it is generally desirable to seek out astrophysical sites which have been the least affected by this. In particular, since metallicity is generally expected to be correlated with the degree of stellar processing that has taken place [53,54], sites with low metallicity are thought to be good targets for observation. It is then hoped that, with a sufficient understanding of the intrinsic physics of such sites, one can extrapolate to zero metallicity to obtain the primordial values. The best current observational estimates of the cosmic primordial ^4He abundance are believed to come from studies of helium and hydrogen recombination lines in low metallicity clouds of ionized hydrogen, so-called HII regions, which reside in blue compact galaxies [55].

Presently, however, estimates of the primordial ^4He abundance, extrapolated from observations of these regions by the different groups of observers, exhibit central values that differ by significantly more than their quoted statistical errors. This situation is generally believed to be indicative of the fact that some or all of these estimates are dominated by systematics [56], though it remains possible that the scatter could also be evidence for a genuine variance in the primordial values themselves. In light of this general uncertainty concerning the value of Y_p , we shall adopt here the compromise value, derived from re-analysis of the data by Olive *et al.* [48], who proposed that

$$Y_p = 0.238 \pm 0.002 \pm 0.005. \quad (68)$$

⁸It further follows that, as the baryon density was sensitive to the expansion rate of the Universe, any variation in the expansion rate would also have influenced the final distribution of the abundances.

Here the first error is statistical while the second represents an estimate of the overall systematic uncertainty in modelling the physics of the HII regions. The errors are compatible

with all the data reviewed. In the context of SBBNS, the above estimate of Y_p alone is then consistent with $1.2 \leq \eta \leq 6.3$.⁹

As first indicated by Zel'dovich *et al.* [38] the situation just described could have been radically different if one allowed for the possibility of a population of PBHs, which were hot enough to evaporate nucleons during nucleosynthesis, i.e., PBHs with $T_{\text{BH}} \geq 2$ GeV, since in such an eventuality it turns out (as we shall outline below) that n/p would have continued to increase after the weak interaction freeze-out time, giving rise to the possibility of a significantly higher yield of ${}^4\text{He}$. Based on observational data of the time Zel'dovich *et al.* calculated that an increase in n/p of more than about 80% (from its presently accepted value), would have resulted in an unacceptable overproduction of primordial ${}^4\text{He}$.¹⁰ Consequently they were able to put a constraint on the mass fraction of PBHs evaporating at the time of nucleosynthesis and also to translate this into a constraint on the initial mass fraction.¹¹ A numerical treatment of the effects suggested by Zel'dovich *et al.*, which involved integrating the system of nucleosynthesis reaction equations modified by appropriate terms to account for the (annihilation and spallation) effects of injected PBH baryons, was later carried out by Rothman and Matzner [42]. The results of these simulations broadly concurred with the semianalytical estimates of Zel'dovich *et al.* and provided both an improved quantitative and qualitative understanding of the PBH effects they had suggested.

More recently, however, it has been argued that the majority of baryons emitted by PBHs would not be emitted directly as nucleons and mesons, as Zel'dovich *et al.* had assumed, but rather via the fragmentation of a QCD quark-gluon jet [29]. An analysis of this mode of injection, which utilizes the formalism of Reno and Seckel [57], has been carried out by Kohri and Yokoyama [43]. Attempting to take proper account of QCD effects in this way, they find constraints on the mass-fraction in PBHs that are typically one or two orders of magnitude stronger.¹² An alternative proposition, made by Heckler [58,59], however, is that the QCD jets would not directly fragment into hadrons. Instead, the particles emitted by a PBH would form a dense plasma,

through which the quarks and gluons would lose energy via QCD bremsstrahlung and pair production, and this in turn would give rise to a photosphere, which could then be constrained by observations.

However, since we are here interested in only obtaining a first semi-analytic estimate of PBH effects in the braneworld case, we shall ignore such QCD effects and follow the original approach of Zel'dovich *et al.* [38], keeping in mind its drawbacks and limitations. In order to reconsider this constraint for braneworld primordial black holes, we shall start by estimating the number of particles evaporated by a braneworld PBH during nucleosynthesis. The total number density of emitted particles, N_{em} , resulting from the complete evaporation of a population of PBHs of some given initial mass, may be expressed as¹³

$$N_{\text{em}} = \frac{\rho_{\text{PBH}}}{\langle E_{\text{em}} \rangle}, \quad (69)$$

where $\langle E_{\text{em}} \rangle$ is the average energy of the emitted particles. The ratio of the energy density in PBHs at evaporation to the background radiation energy density, $\rho_{\text{PBH}}/\rho_{\text{rad}}$, is therefore [38]

$$\alpha_{\text{evap}} = \frac{\langle E_{\text{em}} \rangle N_{\text{em}}}{\langle E_{\text{rad}} \rangle N_{\text{rad}}}, \quad (70)$$

where $\langle E_{\text{rad}} \rangle$ and N_{rad} are similarly the average energy and number density of the particles comprising the background cosmological radiation fluid.

To a good approximation, the ratio of the average energies above is given by the ratio of the PBH temperature at the onset of evaporation to the background temperature at evaporation. For the four-dimensional PBHs of standard cosmology, using Eqs. (13) and (14) and applying the standard cosmological temperature-time relation

$$\frac{t}{t_4} = \left(\frac{45}{16\pi^3} \right)^{1/2} g_{\cos}^{-1/2} \left(\frac{T}{T_4} \right)^{-2}, \quad (71)$$

(see, e.g., Ref. [45]) the total emitted number density at any time $t = t_{\text{evap}}$ during nucleosynthesis can be expressed as [38]

$$N_{\text{em}} \approx \alpha_{\text{evap}} \times 0.13 \times g_{\text{brane}}^{1/2} g_{\cos}^{-1/4} \left(\frac{M_i}{M_4} \right)^{-1/2} N_{\text{rad}}. \quad (72)$$

In the braneworld scenario, using Eqs. (5) and (11) results in

$$N_{\text{em}} \approx \alpha_{\text{evap}} \left(\frac{320}{\pi} \right)^{1/4} \tilde{g}^{1/2} g_{\cos}^{-1/4} \left(\frac{M(t_c)}{M_4} \right)^{-1/2} N_{\text{rad}}. \quad (73)$$

Evidently, Eqs. (73) and (72) have the same functional form. However, it is important to recall that both the mass and temperature of PBHs of a given lifetime are reduced in the

⁹Taking account also of the current observational estimates of the other primordial abundances yields the smaller “concordance interval” of $2.6 \leq \eta_{10} \leq 6.2$ [49].

¹⁰Zel'dovich *et al.* took unacceptable to mean $Y_p \geq 0.4$.

¹¹In addition to studying the effects of PBH injected baryons on the value of n/p , Zel'dovich *et al.* also studied the effects of helium spallation by PBH baryons. This they argued would have led to an increase of the deuterium abundance; a claim that was later supported by the numerical studies of Vainer *et al.* [39] and Rothman and Matzner [42]. However, we shall not discuss this effect here.

¹²We note that this analysis could in principle be easily extended to the present context. Moreover, in contrast to the case of standard PBHs, given the somewhat lower Hawking temperature of braneworld PBHs evaporating at nucleosynthesis, such an analysis would not entail a large extrapolation of the behavior of QCD from presently observed experimental regimes.

¹³Note: In this section we use a notation in which N_{em} is a number density and we reserve n specifically to mean the neutron number density.

braneworld scenario. Taking the mass reduction in account, Eqs. (72) and (73) show that the emitted number density for a given mass fraction α_{evap} is increased in the braneworld scenario. But this is obvious from the temperature reduction, which reduces the average energy $\langle E_{\text{em}} \rangle$ per emitted particle.

Having found an estimate of the total number density of particles evaporated, we now turn our attention to the interaction of these particles with the cosmological background [38]. In the simplest case (i.e. in absence of any PBH chemical potentials) one expects PBHs to emit nucleons and anti-nucleons with equal measure. First let us consider the case of the neutrons and anti-neutrons. An anti-neutron emitted by a PBH has two possible fates; it may either annihilate with a background neutron, or alternatively annihilate with a background proton. At high momenta the cross sections for these processes are essentially the same. The first of these possibilities leads to no net change in the neutron-to-proton ratio. This follows from the fact that on average for every anti-neutron emitted by a PBH there is also a neutron emitted, thus the background neutron which is annihilated will in effect only be replaced on average by another neutron also emitted by the PBH. On the other hand, by the same reasoning, in the second interaction the background proton is effectively replaced with a PBH neutron. Moreover, because there are six times as many protons in the cosmological background as neutrons, this latter reaction is six times more likely than the former. Hence, on average we expect that *for every seven neutrons emitted, one simply replaces a background neutron and the other six replace background protons*, thus increasing the neutron-to-proton ratio.

The story for the protons and anti-protons emitted by the PBH is similar. An anti-proton emitted by a PBH may either annihilate with a background proton, or with a background neutron. Here again it is apparent that the first case leads to no net change in the neutron-to-proton ratio, as the annihilated background proton will in effect just be replaced by a PBH proton, whereas in the second interaction, a background neutron is effectively replaced with a PBH proton. In this case, however, the latter reaction is six times *less* likely than the former. Hence, we expect that *for every seven protons emitted, six simply replace background protons while only one replaces a background neutron*.

To summarize, on average every seven anti-neutrons and seven anti-protons emitted by an evaporating PBH (along with equal numbers of their anti-particles) effectively convert twelve background protons and two background neutrons into seven protons and seven neutrons. Hence, the change in the background neutron number density, n_c , is

$$\delta n_c = \frac{6}{7} n_{\text{em}} - \frac{1}{7} p_{\text{em}} \approx \frac{5}{7} n_{\text{em}}, \quad (74)$$

where n_{em} and p_{em} are respectively the number densities of the neutrons and protons emitted by evaporating PBHs, which to a good approximation will be the same. Similarly, we find that $\delta p_c \approx -(5/7)n_{\text{em}}$, so that $\delta(n+p)=0$.

As discussed above, current observational estimates suggest that the ^4He abundance, Y_p should be $23.8 \pm 1.1\%$, where we have added the errors in quadrature and quoted the

2σ value. Assuming this to be a legitimate conservative estimate, then it obviously follows that the largest value that could at present be accommodated by a nucleosynthesis scenario that included the effects of PBH evaporations must be 24.9% . The question that we wish to answer, however, is how much could PBH effects have actually contributed to Y_p . To ascertain this we use an independent piece of information, namely the lower bound on η from observations of $\text{Ly}\alpha$ absorption in quasar spectra [60], which states that $\eta_{10} \gtrsim 3.4$. If we take this bound at face value, then since SBBNS conserves η and PBH evaporations decrease it, it follows therefore that nucleosynthesis could not have started at a lower value of η than this. Now for this value of η SBBNS predicts a value of $Y_p \gtrsim 24\%$. Moreover, since nucleosynthesis with PBHs will always produce a higher value of Y_p , it is impossible to have a nucleosynthesis scenario which incorporates PBH effects for $\eta=3.4$ such that Y_p will be less than this. Thus a necessary condition is that PBHs could not have increased the value of Y_p by more than about 0.9% . Hence, we may estimate that PBHs evaporating during nucleosynthesis may only increase Y_p by as much as about 1% , or equivalently that

$$\delta \left(\frac{2n}{n+p} \right) = \frac{2\delta n}{n+p} \approx \frac{5}{7} \frac{2n_{\text{em}}}{n+p} < \frac{1}{100}. \quad (75)$$

For all presently allowed values of l , we find that PBHs which evaporate during nucleosynthesis emit nucleons. In general, however, it is not known exactly what proportion of the particles injected into the background by the PBH will constitute nucleons. In the standard scenario, however, Carr [11] has estimated that around 20% of the particles emitted by a PBH decaying during nucleosynthesis will ultimately go into nucleons and anti-nucleons. Here, therefore, we shall assume that the PBHs will emit a fraction $\mathcal{F}N_{\text{em}}$ of the total emitted particles in nucleons and anti-nucleons, and assume similarly that $\mathcal{F} \lesssim 0.2$. Therefore, since $n_{\text{em}} \approx \mathcal{F}N_{\text{em}}/4$, it follows from Eq. (75) that we must have

$$\frac{N_{\text{em}}}{n+p} \sim \frac{N_{\text{em}}}{n_b} < \frac{2.8}{100\mathcal{F}}. \quad (76)$$

Substituting for N_{em} using Eq. (72) gives the standard constraint [38]

$$\alpha_{\text{evap}} < \frac{0.22}{\mathcal{F}} \left(\frac{g_{\text{cos}}}{g_{\text{brane}}^2} \right)^{1/4} \left(\frac{M_i}{M_4} \right)^{1/2} \eta_{\text{evap}}, \quad (77)$$

where $\eta_{\text{evap}} = n_b/N_{\text{rad}}$, is the baryon-to-photon ratio at evaporation. If one further assumes that η is fixed from evaporation to the present day,¹⁴ i.e. $\eta_{\text{evap}} = \eta_0$, this may also be written as

¹⁴Strictly such an assumption is only applicable in absence of further PBH evaporations after nucleosynthesis, as these would contribute radiation to the background and so lower the baryon-to-photon ratio [61]. However, even if this were to occur, such a change is in any case constrained to be fairly small and so should not affect our result much.

$$\alpha_{\text{evap}} < 6 \times 10^{-9} \frac{\Omega_b h^2}{\mathcal{F}} \left(\frac{g_{\text{cos}}}{g_{\text{brane}}^2} \right)^{1/4} \left(\frac{M_i}{M_4} \right)^{1/2}, \quad (78)$$

where we have used the relation, $\eta_0 \approx 2.8 \times 10^{-8} \Omega_b h^2$.

Carrying through the same calculation for braneworld PBHs we find

$$\alpha_{\text{evap}} < 3 \times 10^{-10} \frac{\Omega_b h^2}{\mathcal{F}} \left(\frac{g_{\text{cos}}}{\tilde{g}^2} \right)^{1/4} \left(\frac{M(t_c)}{M_4} \right)^{1/2}. \quad (79)$$

Substituting the factors $\tilde{g} = 0.1$, $g_{\text{brane}} = 106.75$, $g_{\text{cos}} = 10.75$, $\mathcal{F} = 0.2$ and $\Omega_b h^2 \approx 0.02$, the observational constraints finally are written as [38]

$$\alpha_{\text{evap}} < 1.1 \times 10^{-10} \left(\frac{M_i}{M_4} \right)^{1/2} \equiv L_{\text{evap}}^{\text{ST}} \quad (80)$$

and

$$\alpha_{\text{evap}} < 1.7 \times 10^{-10} \left(\frac{M(t_c)}{M_4} \right)^{1/2} \equiv L_{\text{evap}}^{\text{HE}}. \quad (81)$$

Making contact with Sec. III, the ratio of the upper limits can be expressed as

$$\frac{L_{\text{evap}}^{\text{HE}}}{L_{\text{evap}}^{\text{ST}}} \approx \left(\frac{l}{l_{\text{min}}} \right)^{-1/4}, \quad (82)$$

with $l_{\text{min}}/l_4 \approx 10^{14} - 10^{15}$ for PBHs evaporating during nucleosynthesis. By assumption $l > l_{\text{min}}$, hence the constraint will be tighter in the braneworld case. This can be understood as follows: The change in helium abundance due to evaporating PBHs is proportional to the emitted number density, see Eq. (75). But for a given mass fraction α_{evap} , this number density is increased for braneworld PBHs, as alluded to before. Figure 3 shows how the constraints are modified as compared to the standard scenario.

Using the expressions of Sec. III, the bounds imposed at nucleosynthesis can be converted into bounds on the initial PBH mass fractions α_i . For standard cosmology this gives [38]

$$\alpha_i < 3 \times 10^{-18} \left(\frac{M_i}{10^9 \text{ g}} \right)^{-1/2}, \quad (83)$$

for PBH masses in the range $10^9 \text{ g} < M_i < 10^{10} \text{ g}$. In the braneworld scenario we obtain

$$\alpha_i < 4.79 \times 10^{-11} \left(\frac{l}{l_4} \right)^{(1-7q)/4(4-q)} \left(\frac{t_{\text{evap}}}{t_4} \right)^{3(q-1)/4(4-q)}, \quad (84)$$

with t_{evap} ranging from 1 to 400 s.

For the limiting case where $l = l_{\text{max}} = 10^{31} l_4$, neglecting accretion ($q = 0$), and expressing the bound in terms of the initial PBH mass M_i this gives

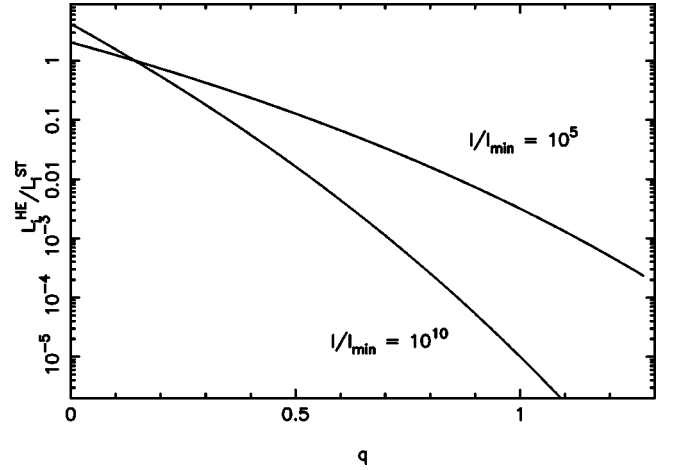


FIG. 3. The behavior of Eq. (30) as a function of the accretion parameter q for two choices of the ratio l/l_{min} , as appropriate to the helium constraint. This is to be compared with Fig. 2, as the imposed limits at evaporation in the high energy and standard treatments, $L_{\text{evap}}^{\text{HE}}$ and $L_{\text{evap}}^{\text{ST}}$, are no longer equal. Their ratio is instead given by Eq. (82). In the case of the helium constraint, the limits are more strongly tightened in the high-energy regime than for other constraints. Note that for the highest values of q , l may not be allowed to be quite as large as l_{max} .

$$\alpha_i < 3 \times 10^{-17} \left(\frac{M_i}{10 \text{ g}} \right)^{-3/8}, \quad (85)$$

with M_i in the range $10 \text{ g} < M_i < 200 \text{ g}$.

As remarked in Sec. II, for extreme values of the AdS radius combined with a highly efficient accretion process, PBHs that are to evaporate during the nucleosynthesis era would have been too light to be consistent with the lower mass limit imposed by inflation. We therefore end with an example where accretion is efficient ($q = 4/\pi$), but take a more moderate value for the AdS radius, $l = 10^{25} l_4$. The initial mass fractions are then constrained as

$$\alpha_i < 6 \times 10^{-26} \left(\frac{M_i}{10^{-4} \text{ g}} \right)^{0.102}, \quad (86)$$

for initial masses with range $10^{-4} \text{ g} < M_i < 10^{-2} \text{ g}$.

B. Deuterium photodisintegration constraint

The high-energy particles emitted by evaporating PBHs both during and after nucleosynthesis can be sufficiently energetic to disrupt primordial nuclei. One important reaction of this type is so-called *photo-disintegration*, or in other words, the destruction of primordial nuclei by high-energy PBH photons. Of all the primordial nuclei deuterium is the most susceptible to photo-disintegration, since it has both the highest cross section and also the lowest threshold, $Q_d \sim 2.25 \text{ MeV}$. A detailed analysis of this effect was considered by Lindley [41] for the case of standard four-dimensional PBHs. Here we briefly review that work and extend the analysis to the context of the RS-II cosmology.

Evaporation products may interact with the background up until recombination. Therefore, in order to consider the effects of PBH photo-disintegrations on the light element abundances one is generally concerned with PBHs evaporating between the end of nucleosynthesis at 400 s and the time of recombination at $t_{\text{rec}} \approx 10^{12}$ s. For standard cosmology, these are PBHs with masses in the range

$$10^{10} \text{ g} \leq M_i \leq 10^{13} \text{ g} \quad (87)$$

and temperatures in the range

$$10^3 \text{ GeV} \geq T_{\text{BH}} \geq 1 \text{ GeV}. \quad (88)$$

In the braneworld case both ranges will be reduced. For the extreme case when $l = l_{\text{max}}$, the ranges become

$$200 \text{ g} \leq M(t_c) \leq 10^7 \text{ g} \quad (89)$$

and

$$200 \text{ MeV} \geq T_{\text{BH}} \geq 1 \text{ MeV}. \quad (90)$$

Although high-energy PBH photons are directly capable of causing photo-disintegration of nuclei, such *direct* disintegrations are in fact extremely rare, since the cross sections for photo-nuclear reactions, even for deuterium are extremely small, typically $\leq 10^{-6}$. The main effect rather is indirect and comes instead from the photo-disintegrations caused by the very large number of lower energy photons which are produced as a consequence of the thermalization of the high-energy PBH photons with the background. This thermalization process may be understood as follows. The high-energy photons emitted by the PBHs interact with the background via two main processes, namely via Compton scattering off the background electrons and via electron-positron pair production off the nuclei. The energetic electrons and positrons produced in these processes then in turn subsequently predominantly lose energy via inverse Compton scattering off the background photons. These photons then again Compton scatter and pair-produce and so on. In this way a single high-energy PBH photon gives rise to a “cascade” of photons, electrons and positrons of increasing number and decreasing energy.

The set of photons in each cascade, with energies $\{E_i\}$, above the threshold Q_d will form a finite set, and although these photons will all predominantly interact via Compton scattering and pair-production processes, nevertheless will have a small probability $P(E_i)$ of destroying a deuteron. This probability is given by the ratio l_γ/l_d of the mean free path of the deuteron to the photon. If one ignores the tiny fractions of ^3He , Li and D , the mean free path of a photon of energy E in the background is approximately [41]

$$l_\gamma^{-1} \approx n_e \sigma_c + (n_H + 4n_{He}) \sigma_{\text{pp}}, \quad (91)$$

where σ_c and σ_{pp} are respectively the Compton scattering and pair-production cross sections. Since charge neutrality demands that $n_p = n_e$, this may be approximated by l_γ^{-1}

$\approx n_p \sigma_t$, where the “total cross section” $\sigma_t \approx \sigma_c + \sigma_{\text{pp}}$. Hence, the probability of destroying a deuteron is given by [41]

$$P(E_i) = \frac{n_d \sigma_d(E_i)}{n_p \sigma_t(E_i)}, \quad (92)$$

where n_d and n_p are the number densities of the deuterons and protons, and σ_d is the $d(\gamma, n)p$ cross section. Hence, summing over all the photons in a single cascade gives the number of deuterons destroyed, i.e.

$$dN_d = \sum_i P(E_i). \quad (93)$$

In reality, however, each cascade is different. In order to take account of this variance one may introduce an average distribution $N(E', E)$, so that the average number of deuterons destroyed by a high-energy PBH photon with initial energy E is then given by [41]

$$dN_d = - \int_{Q_d}^E N(E', E) P(E') dE'. \quad (94)$$

In practice the form of $N(E', E)$ can be determined by numerically modelling many cascades. Adopting such an approach, Lindley was able to empirically estimate¹⁵ the above integral to be of the form [41]

$$dN_d = - \frac{n_d}{n_p} \beta \left(\frac{E}{E_*} \right). \quad (95)$$

Here E_* and β are constants, with $E_* \approx 10^{-1} \text{ GeV}$ and $\beta \sim 1$.

Thus far we have focussed only on the effect of photo-disintegrations caused by high-energy photons of a single energy E . However, PBHs will emit photons with a spectrum of energies. Let us consider a comoving volume V , containing a population of PBHs with total mass $M(t)$. In a time dt it will evaporate a fraction $f_\gamma dM$ of its mass into photons with a spectrum of energies $\nu(E)$, such that

$$\int E \nu(E) dE = -f_\gamma dM. \quad (96)$$

The number of deuterons in the volume V destroyed in this time will therefore be given by integrating over the spectrum of cascades that these evaporating photons will give rise to, i.e.

$$dN_d = - \int_{Q_d}^E N(E', E) P(E') dE' \nu(E) dE. \quad (97)$$

Using Eqs. (95) and (96) we can easily integrate this last expression to give [41]

¹⁵Lindley also assumed that n_d/n_e did not change much over the thermalization timescale, which is consistent with the underlying presumption that n_d could not have changed too much.

$$\frac{N_d(t_2)}{N_d(t_1)} = \frac{X_d(t_2)}{X_d(t_1)} = \exp\left(-\frac{\Delta M f_\gamma \beta}{N_p E_*}\right), \quad (98)$$

where $\Delta M = M(t_1) - M(t_2) > 0$ is the PBH mass evaporated between times t_1 and t_2 , $N_d(t_i)$ is the deuteron number contained in V at time t_i and $X_d = n_d/n_b$ is the deuteron baryon fraction. In principle, Eq. (98) should strictly apply when PBH evaporation is the *only* process responsible for changing N_d , and should therefore apply to the time interval between the end of nucleosynthesis and recombination, i.e. from ~ 400 s until $\sim 10^{12}$ s.¹⁶

Between the times t_1 and t_2 deuterons are destroyed, so obviously $X_d(t_2) < X_d(t_1)$. In addition, in order to be consistent with observational limits on the D abundance, the depletion of X_d cannot be too large. Hence, we need to demand that $X_d(t_2) > e^{-\epsilon} X_d(t_1)$ say, where $\epsilon > 0$ is some number of order unity to be constrained by observation.¹⁷ With this requirement we therefore have the condition

$$0 < \frac{\Delta M f_\gamma \beta}{N_p E_*} \lesssim \epsilon. \quad (99)$$

Taking $N_p \approx M_b/m_p$, where M_b is the baryonic mass and m_p the proton mass, then [41]

$$\frac{\Delta M}{M_b} \lesssim \frac{\epsilon}{f_\gamma \beta} \frac{E_*}{m_p}. \quad (100)$$

To translate Eq. (100) into a bound on some mass fraction of PBHs, we need to make an assumption about the PBH mass spectrum. It is possible PBHs only form in a narrow mass range and equivalently only evaporate at a specific era. Then t_1 and t_2 are simply taken to contain that era and Eq. (100) is effectively a constraint on the total PBH mass fraction of the model. On the other hand, PBHs could exhibit an extended, smoothly varying spectrum. In this case the total mass evaporated is dominated by the low mass end of the spectrum, i.e. by those PBHs evaporating earliest. We may take t_1 and t_2 to be the end of nucleosynthesis and the onset of recombination respectively, and estimate ΔM by the PBH mass evaporated shortly after nucleosynthesis. It is therefore usually justified to take

$$\frac{\Delta M}{M_b} \approx \left[\frac{\rho_{\text{PBH}}}{\rho_b} \right]_{t_{\text{evap}}}, \quad (101)$$

with t_{evap} some time after nucleosynthesis, either when a narrow mass range of PBHs evaporates, or straight after nucleosynthesis for an extended mass spectrum. Eq. (100) is then equivalent to

¹⁶In fact the range of validity can be extended back to the beginning of nucleosynthesis [41], but we shall not consider this here.

¹⁷Here Lindley chooses a rough bound of $\epsilon \sim 1$, corresponding a decrease in X_d of one e -fold over the period $t_2 - t_1$.

$$\alpha_{\text{evap}} \lesssim \left[\frac{\rho_b}{\rho_{\text{rad}}} \right]_{t_{\text{evap}}} \frac{E_* \epsilon}{m_p f_\gamma \beta}. \quad (102)$$

Furthermore, since $\rho_b \propto a^{-3}$ and $\rho_{\text{rad}} \propto a^{-4}$, it follows that

$$\left[\frac{\rho_b}{\rho_{\text{rad}}} \right]_{t_{\text{evap}}} = \frac{a(t_{\text{evap}})}{a(t_{\text{eq}})} \left[\frac{\rho_b}{\rho_{\text{rad}}} \right]_{t_{\text{eq}}} \approx 2 \frac{a(t)}{a(t_{\text{eq}})} \Omega_b(t_{\text{eq}}), \quad (103)$$

as $\rho_{\text{rad}} = \rho_{\text{tot}}/2 \approx \rho_c/2$ at the time of matter-radiation equality, $t_{\text{eq}} \approx 8 \times 10^4 t_4$. The baryon density parameter at equality is readily related to the present one, as the matter density parameter at equality is given by $\Omega_m(t_{\text{eq}}) \approx 1/2$:

$$\Omega_b(t_{\text{eq}}) = \frac{\Omega_m(t_{\text{eq}})}{\Omega_m(t_0)} \Omega_b(t_0) \approx \frac{1}{2} \frac{\Omega_b(t_0)}{\Omega_m(t_0)}. \quad (104)$$

It follows that Eq. (102) may also be written as

$$\alpha_{\text{evap}} \lesssim \frac{E_* \epsilon}{m_p f_\gamma \beta} \left(\frac{t_{\text{evap}}}{t_{\text{eq}}} \right)^{1/2} \frac{\Omega_b(t_0)}{\Omega_m(t_0)}. \quad (105)$$

As a rough estimate for the fraction of the PBH mass that decays into photons we take $f_\gamma = 0.1$, while the ratio of the present baryonic density to the total matter density will be taken to be 0.1. A bound on the size of the depletion factor, ϵ , can be given by taking the difference between the lowest allowed observational value and the highest allowed theoretical value of D from SBBNS (without PBHs), i.e. assuming the validity of the quasar Lyman- α bound of $\eta \geq 3.4$ as before. Following Steigman [62], a cautious current observational bound on the deuterium abundance is $D/H = 3.0^{+1.0}_{-0.5} \times 10^{-5}$. On the other hand, a value of $\eta \geq 3.4$ implies a maximum SBBNS value of $D/H \leq 7.0 \times 10^{-5}$. Thus, we may take $\epsilon \sim 1$, as did Lindley. Filling in all the parameters then finally leads to the observational constraint

$$\alpha_{\text{evap}} \lesssim 3.5 \times 10^{-29} \left(\frac{t_{\text{evap}}}{t_4} \right)^{1/2} \equiv L_{\text{evap}}. \quad (106)$$

It should be noted that the constraint applies to small five-dimensional PBHs produced in the high-energy regime as well as to conventional four-dimensional PBHs. This is because the amount of deuterium destroyed will be proportional to the total amount of emitted energy, see Eq. (98). For a given mass fraction α_{evap} the emitted energy is identical by definition.

As in the previous sections, we convert Eq. (106) into a constraint on an initial PBH mass fraction, and will take $t_{\text{evap}} = 400$ s. In standard cosmology this gives

$$\alpha_i < 10^{-21}. \quad (107)$$

For braneworld black holes, taking $l = l_{\text{max}}$ and $q = 0$, we find

$$\alpha_i < 10^{-16}. \quad (108)$$

Finally, if $l = 10^{25} l_4$ and $q = 4/\pi$, the initial constraint reads

$$\alpha_i < 4 \times 10^{-26}. \quad (109)$$

VIII. CONCLUSIONS

If there were sufficiently late periods of black hole formation in the early Universe, their presence could be noticeable in more recent cosmological epochs such as nucleosynthesis and beyond. Observation puts upper bounds on these effects and therefore on the allowed abundance of PBHs, conventionally expressed as an upper limit on the total PBH energy fraction. Using the evolution equations for the black holes and the background cosmology, the observational constraints can be translated into constraints on the PBH formation rate. The initial constraints are usually the most severe for those black holes with lifetimes comparable with the cosmic time of the epoch at which the observational constraint is imposed. If PBHs form from the collapse of background density perturbations, their formation rate is related to the amplitude of the power spectrum, on scales that enter the Hubble horizon around the time the PBHs form. Therefore, an initial constraint implies an upper limit to the power spectrum on that scale.

In the RS-II cosmology that we have considered, PBHs of a given lifetime t_{evap} would have formed in the high-energy regime, provided the AdS radius of curvature l is larger than $l_{\text{min}} \propto (t_{\text{evap}}/t_4)^{1/3} l_4$. Then the black hole's mass and temperature at the onset of evaporation are reduced, and observational constraints have to be adjusted in some cases. In addition, and in contrast to four dimensional cosmology, the black hole is likely to grow by accretion of the cosmic background as long as it is in the high energy phase. Care is needed in this matter, as the growth depends very sensitively on the efficiency of accretion. Such black holes will be small and effectively 5D throughout their lifetime.

We see that both the PBH and background evolution can be altered compared to standard cosmology. As a consequence, the translation of an observational limit into an upper limit on the initial PBH mass fraction will be modified, see Eq. (30). Most of the effects to be constrained are simply proportional to the total energy in PBHs, rendering the observational constraints in standard or braneworld cosmology identical by definition. Equation (30) then allows a comparison of the strength of the initial constraint in both scenarios. The braneworld constraint will be the weaker if the accretion efficiency is below 50%, whilst being stronger for accretion efficiencies above 50%, for all values of the AdS radius $l > l_{\text{min}}$. It should be noted that in the latter case the initial mass of the PBHs is also much smaller than in the standard treatment, so that the constraint corresponds to perturbations on smaller scales.

If PBHs were sufficiently heavy to have survived to the present day, their mass density should not exceed that of dark matter. For PBHs with lifetimes marginally exceeding $t_0 \approx 8 \times 10^{60} t_4$, this implies a constraint on the initial PBH mass fraction α_i . If the PBHs formed in a standard cosmological regime, the bound reads $\alpha_i < 10^{-18}$. If they were formed in the high-energy regime, taking the example of

$l = l_{\text{max}} \approx 0.1$ mm and 100% accretion efficiency, the bounds strengthens to $\alpha_i < 10^{-23}$.

Primordial black holes evaporating between photon decoupling and the present age leave behind a spectrum that peaks at a temperature of order the black hole temperature at the onset of evaporation of PBHs with $t_{\text{evap}} \approx t_0$. Focusing on the photon component, it is required that its density is less than the diffuse cosmic background at comparable temperatures. In standard cosmology the peak temperature lies around 100 MeV, while for $l = l_{\text{max}}$ it can be as low as 200 keV. In all cases, the photon spectrum constraint implies bounds of roughly 9 orders of magnitude stronger than the dark matter constraint.

If evaporation products are released around the Sunayev-Zel'dovich time $t_{\text{SZ}} = 10^{-10} t_0$, they will fail to fully thermalize with the background radiation. This time, however, is sufficiently early in order for the excess energy to distort the background blackbody spectrum. Limits on the allowed distortion of the CMB spectrum then imply limits on PBH mass fractions. In standard cosmology one obtains $\alpha_i < 10^{-21}$, while the extreme braneworld case ($l = l_{\text{max}}$, 100% accretion) results in $\alpha_i < 10^{-28}$.

If there was a population of PBHs evaporating during or after the era of nucleosynthesis ($\sim 1 - 400$ s), this would have led to changes in the final light element abundances. As a first example, we note that the abundances depend on the neutron-to-proton ratio n/p at the onset of nucleosynthesis. A standard calculation predicts $n/p \approx 1/7$. However, due to PBH evaporation products an approximately equal amount of (anti-) protons and neutrons is injected into the background, increasing the neutron-to-proton ratio. This in turn implies an increase in the helium-four mass fraction Y_p . Comparing the observed value with standard theoretical predictions then leads to a PBH constraint. Furthermore, the increase in Y_p is proportional to the number density of evaporation products. Recall that a PBH of given lifetime has a reduced temperature in the braneworld scenario. Thus, the associated constraint on the mass fraction α_{evap} of PBHs at evaporation is strengthened in the braneworld case, contrary to what might naively be expected. The resulting initial constraint reads $\alpha_i < 10^{-18}$ in standard cosmology.

For the braneworld case, the following must be borne in mind: If there was a period of inflation, an upper limit on its energy scale is imposed by the amount of produced gravitational waves. The mass of subsequently formed PBHs is then bounded from below, in turn implying a lower limit to the PBH's lifetime. Its strength grows with the AdS radius and accretion efficiency, and for extreme values PBH evaporation could not have been effective until long after nucleosynthesis. We have chosen $l = 10^{25} l_4$ and 100% accretion efficiency in the initial constraint imposed by helium-four production [Eq. (84)], to obtain $\alpha_i < 10^{-26}$.

In a second example inspired by nucleosynthesis considerations, it was remarked that evaporation products can be sufficiently energetic to destroy newly formed primordial nuclei. We have focused on photo-disintegration and considered the change in deuterium abundance, since this is the

nucleus most susceptible to disintegration. The amount of deuterium that would be destroyed grows with the total amount of injected PBH energy, showing that the ensuing bound on α_{evap} does not depend on the individual PBH temperature. For the standard initial constraint, one arrives at $\alpha_i < 10^{-21}$. In the example where $l = 10^{25} l_4$ and accretion is maximally efficient, this becomes $\alpha_i < 10^{-26}$.

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